Gradients of gradients

Teacher notes

Why use this resource?

This resource introduces the second derivative through a sketching task using some of the cards from Gradient match. Students are then asked to reflect on how features of the second derivative relate to the original graphs. The pairs of graphs have been chosen because of their contrasting features: points of inflection and the importance of the sign of the second derivative are discussed in the solution.

Four example functions are then used to illustrate these ideas and to offer opportunities to connect graphical and algebraic thinking.

Preparation

If you have already prepared a set of cards from Gradient match then these could be made available to support discussion, but the graphs in question are available on the printable version of the problem, with empty axes for student sketches below them. This layout is intended to encourage students to sketch their second derivative graphs using a similar horizontal scale to the original graphs.

Possible approach

As a first step, students could be asked to give three reasons why the lower graphs show the gradient functions of the graphs above.

When sketching graphs of the second derivatives, students should be encouraged to use the same scale as that used in the given graphs. This will allow for comparison of features, even though it is difficult to say much more than where the second derivative is positive or negative and when it is zero.

Students should be given the opportunity to discuss their observations from the first part of the task before starting the algbraic examples. This should help them to reflect on these ideas when working on the algebra and sketches, rather then simply working through the algebra.

Key questions

- Where will the second derivative be zero?
- Where will the second derivative be positive? negative?
- (Connections with the original functions) Think of the graph as a road on a map. If you were walking or driving along the road, which way would you be turning?

Possible extension

Ask students to look at the other pairs of cards from Gradient match.

- What can they say about the second derivative in each case?
- For each pair, can they say anything about a function that these graphs could be the first and second derivative of?

For another resource looking at graphical features and calculus, try Choose your families.