Thinking constantly

Teacher notes

Why use the resource?

The warm-up reminds students why the constant of integration is important. The questions are open to discussion, as they are not given any initial conditions to work with, so students will have to think about what the constants of integration represent and which mathematical approach provides an accurate answer.

In the main problem, they are asked to consider a kinematics problem. By changing the constants of integration, students can see how the functions and their graphs that describe aspects of the movement change, and how that links the acceleration, velocity and displacement together to describe the movement of the object.

Possible approach

In the main problem, students would ideally have access to some graphing software that allows them to put in constants that can be manipulated (GeoGebra or Desmos work well). Then once they had found the functions for velocity and displacement, they could graph and manipulate them for themselves, in order to answer the questions in the main problem. If this is not possible, then an interactive graph with the main problem below has been provided. Students can work out the two functions (and could try to sketch them), before you project the graph for them to look at and discuss.

Key questions

If students are forgetting the constant of integration in the warm-up then you may wish to ask them:

- What is the acceleration of an object whose velocity is given by $v(t) = 3t^2 - 2t + 4$?
- What is the acceleration of an object whose velocity is given by $v(t) = 3t^2 - 2t - 8$?

Many kinematics problems have a displacement of zero at $t = 0$ so it is important to highlight the difference between the two calculations shown in the warm-up.

- One of these gives us your displacement from your starting point, and one gives us your displacement relative to a fixed origin. Which is which?

In the main problem, there are questions within the solution that could be asked. Many of them focus on how the features of the graph change and what impact that has on the movement of the object. This concrete example provides an opportunity for students to notice various things about the behaviour of a function and its derivative. For example, if the gradient function always positive then the original function has no stationary points.

Possible extension

Students could be asked to create their own acceleration function, to then find the velocity and displacement functions and see if changing the constants has the same impact as in the original. Or students could be asked to create an acceleration function that creates a journey with specific features, for example, the displacement is always negative, but the velocity is not.