What else do you know?

Teacher notes

Why use this resource?

This problem reinforces the importance of considering what functions look like. The resource presents an image of a part of an unknown function. Students are given the area under the curve in a specified region and asked to consider what they can deduce about a selection of related integrals involving transformations of the original function. This is a problem that challenges students to apply geometrical reasoning to something that they might usually only consider algebraically. The subtle difference between the value of the integral and the value of the area under the curve will come out when tackling this problem.

In a more general context this is a lovely problem with which to encourage students' resilience. The minimal information supplied in the image may lead students to think that the problem is very difficult and even impenetrable. However, if encouraged to draw diagrams and reason through each integral geometrically they will be surprised at how far they can get.

Preparation

Students may like to have worked on Between the lines prior to tackling this problem. Providing mini-whiteboards for students to sketch on could be helpful.

Possible approach

It is useful to encourage some quiet, individual thinking time when students first meet this problem. Students could be guided to think about the diagram on its own first, and to write down what they see and what they can know / easily deduce. They might be encouraged to think of any questions that they have about the image. These thoughts can then be shared amongst the class before students attempt to tackle the actual problem in pairs or small groups.

When considering some of the integrals, for example $\int_2^6 (f(x) - 3)\, dx$ it may be useful to share the possible graphical representations illustrated in the Solution with students as a stimulus for discussion.

The particular example $\int_2^6 -f(x)\, dx$ can be useful as an opportunity to talk about the differences between the area under the curve and the value of the integral.

Key questions

Alongside the questions in the resource, you might ask students to reflect on:

- Which integral or integrals did you choose to consider first? Why?
- Is there a difference between the area under the curve and the integral of the function?
- Does it matter what the original function, $f(x)$, looks like?
- How does the y-coordinate of the local maximum affect the graphical representation?

Possible support

Encourage students to sketch possibilities and think about the implications of the changes to the integral.