Binomials are the answer!

Teacher notes

Why use this resource?

This resource explores the idea of using a straight line to approximate a curve at a point and hence find its gradient. Starting with chords to quadratic and cubic curves, the resource then gives a structured algebraic approach to find a general result for the gradient of the curve $y = x^n$.

This can be used as an introduction to the calculus of powers of x. The general result uses the binomial theorem but the special cases can be derived without using the theorem.

Possible approach

Starting with the quadratic and cubic curves, students could pick different values of x as their target points and calculate gradients of chords between x and $x + h$ for a range of values of h. They should then be able to see what the gradient would be at $h = 0$. The class (or groups of students) could then be set to look for patterns or rules that determine the gradient at any given x-value.

If computers or mobile devices are available, you might ask students to use spreadsheets to calculate the gradients of the chords.

The algebraic generalisation is probably best introduced after this numerical work.

Key questions

- Given two points, how do we calculate the gradient of the line between them?
- Given two points on the curve, can we find the gradient of the chord?
- If we change the value of h, what happens to the chord?
- What value of h would give us the gradient of the tangent? What happens if you try to work that out?
- Can you see any patterns emerging?

Possible extension

Students could be asked to look at other curves, both numerically and using the algebraic generalisation:

- Multiples such as $y = 2x^2$
- Additions such as $y = x^2 + x$