Floppy hair

Teacher notes

Why use this resource?

This task is designed to provide opportunity for students to practise differentiating powers of x in the process of solving an interesting looking puzzle. It involves fractional as well as integer powers.

Students use differentiation to locate stationary points on curves and also have to do some manipulation of indices and solve some quadratic and other equations in order to decide which equation goes with which curve. An extra equation is included in order to reduce the opportunities for shortcut by elimination.

Possible approach

Before starting the task, you might show the diagram and ask the class to describe what they see.

- What are the interesting or important features of the curves?
- What do they have in common and how do they differ?

For the main activity, students could be asked to work in pairs or groups of three. They could divide the work between them to reduce the overall time taken, but it would be good to have them check one another's work as there are many potentials for algebraic mistakes.

Key questions

- How could you rewrite that expression to make something you can differentiate?
- Do we know how to differentiate a square root? How else can we write it?
- Some of the expressions involve awkward fractions. Is it necessary to multiply each term by the fraction or could you save yourself some effort?
- What could you do to check your answers? Is there another way to identify which equation goes with which curve?

Possible extension

Some of the curves can be identified by careful examination of their features. Students could usefully think about what a square root or cube root graph looks like near the origin and which is steeper for small values of x. There is symmetry to be observed in at least one curve.

- How many of the curves could you positively identify without doing any calculations?

Students could also be asked to write equations for other curves that meet the original criteria and to sketch them.

- What are the features of the equations that ensure they pass through those points?
- Could you write the equation of a curve that has a very different gradient at the origin?

Possible support

Students may need prompting to expand brackets and express all the terms as powers of x in order to get started. If appropriate, they could be given the quadratic and cubic expressions to do first before working on those involving roots.