Inverse integrals

Teacher notes

Why use this resource?

This resource allows students to find a definite integral of $\ln x$ without knowing how to integrate $\ln x$ directly. It can be used to review knowledge about the inverse functions e^x and $\ln x$, and to discuss how to find the area between a curve and the y axis. This method can be extended to other functions such as $\arcsin x$, once students can integrate $\sin x$.

Students may find Reflecting on change helpful in thinking about inverse functions and differentiation.

Preparation

This resource could be used by students who know the derivative of e^x, but not the integral of e^x. See the videos on the Resources in action page for examples of this. It could also be used by students who already know the integral of e^x, but it would be better if they had not already met the result of $\int \ln x \, dx$.

Possible approaches

The warm-up allows students to notice as much as they can about the two graphs and review what they know about e^x and $\ln x$. Asking students to 'say what they see' is a good habit that they should be encouraged to use in many problems. However, if you wish to give students a more focused question you could ask 'Compare these two graphs, what do you notice?' Important points to draw out are those regarding the inverse nature of the graphs, i.e. the coordinates, the curve and the different areas are all reflected across $y = x$.

Students may naturally start to solve the main problem during the warm-up.

Key questions

For the warm-up the prompt questions are:

- Is there any information you can add to the graphs?
- What links are there between the two diagrams?
- What shapes can you see in the diagrams? Can you work out their areas?

Possible extension

The second problem involving $\arcsin x$ enables students to consolidate their understanding of the graphical links between a function and its inverse.

Students who are confident with algebraic manipulation could think about the general case of $\int_a^x \ln t\, dt$. Can they find a general solution?

You can watch video of students working on this task here.