Rotating derivatives

Teacher notes

Why use this resource?

In this resource, differentiation of $\sin$ and $\cos$ is introduced by considering a point moving round the unit circle. Students use the given diagrams to create a chain of reasoning to explain what the derivatives of $\sin$ and $\cos$ are.

The discussions that follow should start to highlight the interconnected nature of trigonometric functions, their derivatives, vectors and their applications. In particular, this resource offers the opportunity to see the derivative of $\sin\theta$ as $\sin(\theta+\frac{\pi}{2})$ and similarly the derivative of $\cos\theta$ as $\cos(\theta+\frac{\pi}{2})$. Sketching the graphs of e.g. $y=\sin(\theta+\frac{\pi}{2})$, may help students to make further connections.

Preparation

There are two sets of cards that can be printed. The cards from the problem page contain diagrams only, and the cards from the suggestion show the diagrams with prompting questions to help students build their arguments.

Possible approaches

Students could work in small groups to create an explanation using the cards. They could start with the diagram-only cards and after initial thinking time, one or more of the prompt cards could be handed out if groups are struggling to get started or articulate their ideas. Alternatively, students could use the cards with prompting questions from the start if this extra scaffolding would be more beneficial.

The teacher could encourage individual groups to work on different approaches (trigonometry in right-angled triangles or transformations). Alternatively, groups of 4-6 students could be encouraged to try to use all the cards between them to form multiple arguments to answer the problem.

Asking groups or individuals to write down their explanations may help students to bring out key points and to reflect on how they communicate their ideas effectively.

Key questions

- How can you use the speed of P and the size of the circle?
    - How far does P move in t seconds?
    - What angle is swept out by radius OP in t seconds?
- How could you use right-angled triangles?

- How could you use transformations of vectors?
- What does the graph of $y = \sin(\theta + \frac{\pi}{2})$ look like?

Possible support

Less confident students can be supported by the suggestion cards. Remind students of the two key questions from the initial problem.

As a form of support or to focus students' attention on one approach, you could remove either the cards involving Q, or the card with the triangles. This may reduce the complexity of the task and the card(s) could be introduced once students are have constructed an argument for the remaining cards.

Possible extension

Encourage small groups to come up with different arguments and explore connections between these arguments.

Some students may be able to construct an argument without the cards. They could be shown the first diagram in the problem and asked to think about a possible argument, then share their ideas in pairs or small groups. Certain cards could then be used to prompt students to consider alternative approaches.