Trig gradient match

Teacher notes

Why use this resource?

This resource could help students to recognise how their knowledge of transformations of graphs fits with calculus of trigonometry. Students may see how combining these ideas these can help develop their understanding of each.

Preparation

The graphs are available as cards to be cut out and sorted. This includes blank cards for the graphs which need to be sketched. Please note that in Graph cards, graphs 1, 2, 3, 4, 5 show gradient functions and graphs A, B, C, D, E, F show the original functions.

Alternative cards offers an more open task where the labels on these cards do not indicate which functions should be viewed as gradient functions.

Possible approaches

As a starting point, ask students to look at the array of graphs and arrange them according to what they notice. Students may recognise some of the graphs, but also ask them to describe key features and any relationships between the graphs.

When sorting the cards into function and gradient function pairs, encourage students to place the card showing the gradient function below the card showing the original function, so that they can see corresponding features more easily.

When students try to sketch the graph of a function from its gradient function, they may realise that they are sketching one member of a family of functions. Encourage them to compare their sketches to bring out this discussion point.

If students move on to sketching the second derivatives, they may notice that, for some functions, the graphs of $f''(x)$ and $f(x)$ look very similar. The graphs are sketch graphs so it is difficult to determine whether these are the same, but it could be useful to bear this in mind when differentiating these functions using the chain or product rules.

If using Alternative cards, pairs such as F and 2 or B and 1 could be placed either way round, which could be an interesting discussion point. Depending on how much students already know about differentiating trigonometric functions, this could be explored further using Desmos.

Key questions

- What features of the graphs help you match a function with its gradient function?
- Where is the gradient function negative or positive?
- How could you use transformation relationships between the functions to help match up the functions?

Possible support

Here are some smaller tasks which could support an initial discussion of some key ideas.

- Show students cards A and 5. Which is the gradient function of the other?
- Give students cards B, 1, 3 and 4. Which of graphs 1, 3 and 4 shows the gradient function of card B? Ask students to explain why the other cards *can't* show the gradient function of card B.

If students struggle to keep track of which cards are the functions and which the gradients then write a reminder on the board, a mini white board on each table or $f(x)$ and $f'(x)$ could be written on the cards.

Ask more directed questions such as:

- Where is the gradient zero/ positive/ negative? How will that show on the gradient function graph?
- Where is the gradient increasing/ decreasing?
- What happens at the asymptotes?

Possible extension

What would have happened if you hadn't been told which graphs showed the gradient functions? This is equivalent to using the Alternative cards.

Students could be asked

- Do you recognize what functions the graphs represent?
- Can we deduce any rules of differentiation from these graphs?