# Can you find... chain rule edition

Teacher notes

## Why use this resource?

This problem explores the chain rule for differentiating a function of a function. Students are asked to construct functions $f(x)$ and $g(x)$ so that the derivative of $f(g(x))$ satisfies certain constraints. Students will need to work forwards and backwards from these conditions to suitable functions. This problem could be used to consolidate understanding of the chain rule and to develop students' confidence in working strategically with a range of functions. There are opportunities to take the problem further by modifying the given constraints or exploring the difference between necessary and sufficient conditions.

Some parts of Can you find ... curvy cubics edition could be useful preliminary tasks for this resource.

## Preparation

Mini-whiteboards could help to support exploration of ideas.

Providing access to graphing software such as Desmos could help students to explore the effects of composing different functions.

## Possible approach

There is quite a lot to think about at the start of this problem, so it may be helpful to give a few minutes of individual thinking time before students move into pairs or small groups. Encourage students to look at all the problems rather than focusing only on the first one.

Recognising that the chain rule gives us a product is a key idea, since then students can think about what makes a product positive or negative or zero. Another key point is that in the derivative of $f(g(x))$, the derivative of $f$ is evaluated at $g(x)$. Using function notation helps to emphasise this, although students could also tackle this problem using the $\frac{dy}{dx}$ notation.

If students start to think about types of stationary point in part (b), they may attempt to find second derivatives and may not be able to do this for the functions they've found. Instead, encourage them to sketch and think about the sign of the derivative on either side of the stationary point.

There is quite a lot of reasoning involved in coming up with suitable functions for each part of the problem, so asking students to write this down or present it to other students could help them to reflect on their approach.

## Key questions

There are several questions in the suggestion section that could be helpful.

- What can you say about the derivative if a function has a stationary point?
- If $f(g(x))$ has a stationary point when $x = 5$, what could you say about $f'(x)$ or $g'(x)$?
- Could composing functions alter the types of stationary point you have?
- How does the derivative allow you to determine what type of stationary point you have?
- What are some functions with no stationary points? What is the simplest example?

## Possible support

If students are struggling to get started, you could ask them to compose and differentiate a few familiar functions and see what they notice. Recognising the derivative as a product is a key idea, since then they can think about what makes a product positive or negative or zero.

Even if they have already attempted it in the past, some students may like to look at one or more parts of Can you find … curvy cubics edition to help find a way into these problems.

As mentioned above, students may find access to graphing software such as Desmos helpful.

## Possible extension

This task is very open, so asking students to find another example, and another… could be a productive way to reinforce their thinking. By exploring multiple examples of functions, students can start to think about the difference between necessary and sufficient conditions.

Students could also modify the conditions given or suggest some others and see if they can still find examples of functions. For example,

- Can you find $g(x)$ so that $\ln g(x)$ has a maximum?
- Can you find $f(x)$ and $g(x)$ so that $f(g(x))$ has a local maximum where $g(x)$ has a local minimum?
- Can you find $f(x)$ so that $y = 2x + 1$ is a tangent to $y = f(\sin x)$?