Cones

Teacher notes

Why use this resource?

This resources offers a realistic problem relating to familiar school equipment. There are many ways to approach the problem, and part of the problem is to choose an effective one. The more "mathematical" approaches give the opportunity to use some of the following topics (depending upon the approach chosen): areas of sectors, length of arcs, volumes of cones, surface areas of cones, similar triangles and right-angled trigonometry.

We have described one particular approach in detail. It is probably not the most obvious one for students to pick, but it was chosen to highlight a key mathematical idea, namely that we can approximate by ignoring small errors, and in particular that we can approximate the volume of a thin "shell" by multiplying the surface area by the small thickness of the shell. This way of thinking is central to understanding how integration can be used to find volumes and the like.

The Suggestion section provides a structured way to work on the problem through this approach, and the Appendix summarises results on surface areas and volumes of cones, which may be useful for some students.

The result about the curved surface area of a frustum deduced in the solution to this resource is taken up and extended to give the astonishing result in Cutting spheres.

Preparation

It would be very helpful to have some physical sports cones in the classroom, along with rulers and possibly other measuring equipment.

Possible approach

Either show the class a real sports cone or the photograph of them in the resource. Ask them to think about the first question, and then compare methods. Can the class come up with any other methods besides? (The questions in the Possible approaches section may be useful here.)

Then ask the students to work in groups to develop their method to the point that they can give an answer to the original question. If some groups have chosen approaches involving equipment outside of the classroom, ask them to work instead on a solution that is constrained by the limitations of the available equipment.

At the end, compare and discuss approaches. It is also valuable to draw out of the discussion results which may be more widely applicable, such as formulae for volumes or surface areas.

If groups are having difficulty getting started, the Suggestion section provides a structured approach to getting started on two variants of one particular approach.

Key questions

- How can we model the “cone”?
- What measurements can we make, and what will we have to work out?
- What happens if we “open up” a “cone”?

Possible support

The Suggestion section provides a structured questions to help students to make progress, in particular by offering a labelled sketch.

Possible extension

For students who have worked purely numerically, ask for an algebraic solution.