# Elliptical crossings

Teacher notes

## Why use this resource?

This resource gives students some experience of working with the equation of an ellipse as a generalisation of the circle. It also provides practise in calculating midpoints involving algebraic expressions. Recognising structures in the algebra can also enable students to identify the important components in their calculations, saving them time and helping them to generalise their findings. A certain point of view explores a transformations approach to this problem, which sheds much light on the locus of $M$.

## Preparation

Students do not need to have met the equation for an ellipse prior to this resource.

Some familiarity with using simultaneous equations to find points of intersection of graphs will enable students to focus on the bigger ideas, though this resource can also be used to practise this skill.

Giving students access to graphing software while they work on this problem may be helpful.

## Possible approach

Students could be encouraged to consider a value of $c$, of their own choosing, on their own first. After this individual thinking time, students could be asked to work in pairs or small groups to compare their approaches and to consider each other's values of $c$. Pooling their results can then lead on to thinking about the locus of $M$.

## Key questions

- What does the diagram look like for different values of $c$? Can you sketch these on a single copy of the ellipse?
- How can you locate the midpoint of a line segment?
- Are there any values of $c$ that you would not choose? Why?
- *Why* is the locus of $M$ a straight line? (This may lead on to exploring the questions posed in A certain point of view.)

## Possible support

The GeoGebra applet in Things you might have noticed could be shown to help students to visualise the problem.

Asking what would be the easiest value of $c$ to pick might be useful; students may well overlook the possibility of $c = 0$.

## Possible extension

Can you work out the coordinates of $M$ midpoint using the variable $c$ rather than a specific value of $c$? Is this meaningful for all values of $c$?

What happens if you use straight lines of a different gradient instead?