Proving the laws of logarithms

Teacher notes

Why use this resource?

The resource starts with a Warm-up where students are encouraged to generalise some statements about logarithms and observe important connections. They will go on to prove these results in the main parts of the resource. Students are supported to prove the first result or law using a skeleton Proof sort and then adapt this approach to prove the remaining results.

The skeleton proof in the Proof sort section includes some commentary on the steps in the proof as well as formal steps in algebra. There are also blank cards which students could fill in to include some extra steps in the algebra or explain more of the thinking behind the steps.

By working through the proof for themselves students will gain better understanding of where the log laws come from and why they are true.

Preparation

This is a nice follow-up problem to Summing to one.

The cards should be prepared. You may want one set per student depending on the approach taken.

Possible approach

Use of the Warm-up will depend on students' prior experience with logarithms. It may help students to recall log laws that they have already encountered. If students haven't already seen log laws, the relationships between the numbers in these particular examples may give them a way in to seeing a general form. Students can vary the numbers in the examples and use a calculator to test whether the equations still hold, but they should also be encouraged to think about why these results make sense from the definition of a logarithm.

To prove the first generalised result, students may like to try the card sort on their own first and then compare their proofs to see which extra steps others have included. They should be encouraged to question each other to deepen their understanding and help them think about what extra steps could be helpful - should these be algebraic steps or extra explanation?

Generalised versions of the remaining statements from the Warm-up can be found in the Adapting the ideas section, where students are asked to prove these by adapting the approach taken in the proof sort.

Key questions

Warm-up

- Can you describe relationships between the numbers in the equations? Test the result for other numbers that have this relationship or do not have this relationship.
- What if we had general inputs, e.g. a and b instead of 2 and 5?

Proof sort

- What is the result you are trying to prove?
- Do all your steps follow on logically from the previous ones?
- What extra help might someone reading the proof need to move from one statement to the next?
- Do the extra steps you have included read as complete (mathematical) sentences?

Adapting the ideas

- How did the steps in the proof sort help you to prove that result?
- What's the same and what's different about the results you're trying to prove?

Possible support

If using the Warm-up, students may need to be encouraged to think about what it means to generalise a result. The questions at the bottom of the page may support this. For example, what would happen if they changed the 5 in the first equation to a 7? Do they think the equation would still hold? If not, how could they adapt the other numbers to make the equation hold. Use of a calculator or spreadsheet should support these investigations.

Some students might need help in using either words or symbols accurately and helpfully. Thinking about what the word "therefore" is there for will help students to understand its position in a proof.

Possible extension

The students can be asked to adapt the ideas in this proof to help them prove the other laws of logarithms (*Adapting the ideas* section). Students can check each others' proofs and question each other to help decide what steps are needed.

A version of this resource has been featured on the NRICH website. You might like to look at some students' solutions that have been submitted there.