Reach for the stars

Teacher notes

Why use this resource?

This puts an exponential graph alongside some real-world (and some extra-terrestrial) objects. Students can start to tackle the problem using their knowledge of powers of 2. Before long, a calculator is beneficial as the numbers start to get large, but the entire problem can be tackled using trial and improvement with powers of 2. Students who have been introduced to logarithms will find them a useful tool for solving equations more directly, but the resource can equally be used as a pre-logarithm introduction. The problem also involves some interesting unit conversions once we start looking at astronomical data, and just a little bit of Physics.

It could be followed by the review question Can we find bounds for $\log_2 3$?

Preparation

A printed copy of the data could be useful for each group so that the board could be used for other things.

Access to the internet might be helpful for students to research heights and distances of other things.

If possible some A2 or even A3 paper with 1 cm grid lines would be quite helpful to help less confident students.

White boards with grid lines could be useful for starting with.

If display work is wanted as a result some name cards for the different objects could be printed out in advance and an x-axis on a display board ready for it.

Possible approach

Display the problem image and draw students' attention to the scaling of 1cm per unit on each axis. Begin with mini white boards (portrait orientation) ask students to mark along the top edge how far along it the graph of $y = 2^x$ would exit the top of the page and show boards.

Discuss what that x value is and how they know. Show the further questions (these could be printed out) and discuss as a group what they might already know, need to conjecture or research. Possibly show some of the data.

Students could work in small groups making conjectures and doing calculations to confirm. They could come to mark points along an x-axis displayed on the board, or better still a more permanent display.

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Key questions

- How much further along will an object twice as tall be?
- Can you predict what the x-values for other distances would be (length of the Nile, distance from Land's End to John O'Groats, length of the Trans-Siberian Railway)?
- What do you notice?
- What difference would it make to change the 2 to a 3?

Possible support

Students could be encouraged to write a table of values, say as far as $x = 10$.

Grid paper could be useful so that students could plot a graph to start to get a better feel.

Copies of the data from the data tabs could be given early to students who are unable to make size conjectures early on if whole class discussion has not been used.

Possible extension

Students could consider how much difference it would make to change to powers of another base, say 3 or 10.

The Solution section also contains a nice extension idea: "Suppose instead of the graph of $y = 2^x$ we had used $y = x^{10}$, or some other power of x. How different would this diagram be?"

A version of this resource has been featured on the NRICH website. You might like to look at some students' solutions that have been submitted there.

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