Dot product

The *dot product* (or *scalar product*) of the two real vectors

$$\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \quad \text{and} \quad \mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

is

$$\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3.$$

This is a real number (hence the name 'scalar' product). The definition extends to vectors $\mathbf{a}$ and $\mathbf{b}$ with n components. (Compare this to the cross product, which outputs a vector and which does not have such a simple generalisation.)

The dot product combines information about the lengths of $\mathbf{a}$ and $\mathbf{b}$ and the angle between them. Specifically,

$$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}|\,|\mathbf{b}| \cos\theta$$

where $|\mathbf{a}|$ and $|\mathbf{b}|$ are the lengths (magnitudes) of $\mathbf{a}$ and $\mathbf{b}$ and θ is the angle between the two vectors. It follows that for non-zero vectors $\mathbf{a}$ and $\mathbf{b}$, the dot product is zero exactly when $\mathbf{a}$ and $\mathbf{b}$ are at right angles to one another.

For every choice of $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ and every scalar λ, we have

$$\begin{aligned} \mathbf{a} \cdot \mathbf{b} &= \mathbf{b} \cdot \mathbf{a} && \text{(commutativity)} \\ \mathbf{a} \cdot (\mathbf{b} + \mathbf{c}) &= \mathbf{a} \cdot \mathbf{b} + \mathbf{a} \cdot \mathbf{c} && \text{(distributivity over addition)} \\ (\lambda \mathbf{a}) \cdot \mathbf{b} &= \lambda(\mathbf{a} \cdot \mathbf{b}) = \mathbf{a} \cdot (\lambda \mathbf{b}). \end{aligned}$$