

Dot product

The *dot product* (or *scalar product*) of the two real vectors

$$\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \quad \text{and} \quad \mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

is

$$\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3.$$

This is a real number (hence the name 'scalar' product). The definition extends to vectors \mathbf{a} and \mathbf{b} with n components. (Compare this to the [cross product](#), which outputs a vector and which does not have such a simple generalisation.)

The dot product combines information about the lengths of \mathbf{a} and \mathbf{b} and the angle between them. Specifically,

$$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| |\mathbf{b}| \cos \theta$$

where $|\mathbf{a}|$ and $|\mathbf{b}|$ are the lengths (magnitudes) of \mathbf{a} and \mathbf{b} and θ is the angle between the two vectors. It follows that for non-zero vectors \mathbf{a} and \mathbf{b} , the dot product is zero exactly when \mathbf{a} and \mathbf{b} are at right angles to one another.

For every choice of \mathbf{a} , \mathbf{b} and \mathbf{c} and every scalar λ , we have

$$\mathbf{a} \cdot \mathbf{b} = \mathbf{b} \cdot \mathbf{a} \quad (\text{commutativity})$$

$$\mathbf{a} \cdot (\mathbf{b} + \mathbf{c}) = \mathbf{a} \cdot \mathbf{b} + \mathbf{a} \cdot \mathbf{c} \quad (\text{distributivity over addition})$$

$$(\lambda \mathbf{a}) \cdot \mathbf{b} = \lambda(\mathbf{a} \cdot \mathbf{b}) = \mathbf{a} \cdot (\lambda \mathbf{b}).$$