Geometric mean

The *geometric mean* is a type of mean.

To find the geometric mean of a set of n positive real numbers, multiply all of the numbers together and then take the nth root of the product: the geometric mean of a_1, a_2, ..., a_n is

$$\sqrt[n]{a_1 a_2 \cdots a_n}.$$

This is useful when trying to describe an average rate of growth over time. As an example, if something grows by 3% one year, then 2% the next year and 5% in the third year, then it grows by factors of 1.03, 1.02 and 1.05 in the three years. In total, it grows by a factor of

$$1.03 \times 1.02 \times 1.05$$

over the three years. If the growth in each year had been the same "average" growth, say r, then $r^3 = 1.03 \times 1.02 \times 1.05$. Thus r is the geometric mean of the three growth factors.

For positive numbers a_1, a_2, ..., a_n, the arithmetic mean is always greater than or equal to the geometric mean. This is often known as the "AM–GM inequality".

Arithmetic means and geometric means are related. For if we take the logarithm (to any base) of the geometric mean g of a_1, ..., a_n, we find that

$$\log g = \frac{1}{n}(\log a_1 + \log a_2 + \cdots + \log a_n),$$

which is the arithmetic mean of the logarithms of a_1, ..., a_n.