Inverse function

Given a function f, the *inverse* of f is another function g with the property that

$$g(f(x)) = x$$

for every x in the domain of f. If it is also true that

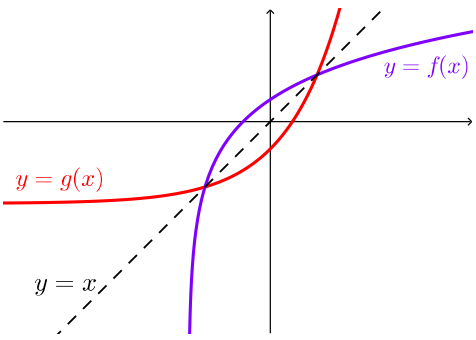
$$f(g(x)) = x$$

for every x in the domain of g then the functions form a pair of inverses.

The inverse function for f is denoted by f^{-1}. The function f has to be a bijection to possess an inverse. The domain of f^{-1} will be the range of f and vice versa.

For example:

- If $f(x) = x + a$, $x \in \mathbb{R}$, then $f^{-1}(x) = x - a$, $x \in \mathbb{R}$.
- The function $f(x) = \cos x$, $x \in \mathbb{R}$ has no inverse, as it is not bijective. However, the function $f(x) = \cos x$, $0 \leq x \leq \pi$ has inverse $f^{-1}(x) = \cos^{-1} x$, $-1 \leq x \leq 1$.
- If $f(x) = e^x$, $x \in \mathbb{R}$, then $f^{-1}(x) = \ln x$, $x \in \mathbb{R}$.



Note that the graph of $y = f^{-1}(x)$ is a reflection of $y = f(x)$ in the line $y = x$.

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