Root mean square



The root mean square is a type of mean.

Given real numbers a_1 , a_2 , ..., a_n , the root mean square (often abbreviated to RMS) is obtained by calculating the arithmetic mean of the squares of a_1 , ..., a_n , and then taking the square root of this:

$$\sqrt{\frac{a_1^2 + a_2^2 + \dots + a_n^2}{n}}$$

It is useful when trying to measure the average "size" of numbers, where their sign is unimportant, as the squaring makes all of the numbers non-negative.

The most common case of using the root mean square is when calculating the standard deviation of a set of numbers $x_1, ..., x_n$. The standard deviation is the root mean square of the deviations of these numbers from the mean, that is, the root mean square of $(x_1 - \bar{x})$, ..., $(x_n - \bar{x})$, where \bar{x} is the mean of $x_1, ..., x_n$, so

standard deviation =
$$\sqrt{\frac{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + \dots + (x_n - \bar{x})^2}{n}}$$

The root mean square can also be used for continuous functions, with integration replacing summation. If the function f(x) is defined for $a \le x \le b$, then the root mean square value of f(x) over this interval is

$$\sqrt{\frac{1}{b-a}} \int_a^b (f(x))^2 \, dx.$$

The root mean square is an example of a power mean.