Mapping a derivative

Teacher notes

Why use this resource?

This resource offers a way to think about differentiation which is very different from the familiar chords-of-a-graph approach: it shows that a function can be thought of as a local scaling, and the derivative is this scaling (or the scale factor of the scaling). This offers several benefits: for example, it offers a different way of thinking about the relationship between displacement and velocity, independent of graphs, and from there into integration (though only the first of these ideas is touched on within this resource); it also offers the tools to give a very convincing argument for why the chain rule for differentiation works, as explored in Chain mapping.

The approach taken in this resource is via mapping diagrams, building on the ideas developed in Mapping a function.

A fuller discussion of the potential power of this approach is discussed in the blog post Percolation, patience and the chain rule.

Preparation

Students will gain the most benefit from this resource if they have already explored linear functions using Mapping a function.

Students will either need to be able to draw several mapping diagrams or to use the GeoGebra applet on the website. Blank mapping diagrams can be downloaded and printed in advance.

Possible approach

Students could first be asked to predict what they think the mapping diagram for $f(x) = x^2$ will look like, both at the normal scale and when zoomed in, before drawing the diagram (by hand or with the applet).

Drawing the mapping diagrams zoomed in at different centre points is likely to provoke some curiosity and surprise. How do these images relate to ones we have seen before, and what does this mean?

A plenary could be used to discuss the relationship between this approach and other approaches to differentiation which students have seen.

Key questions

- How does the function behave when we zoom in at a point?
- How does the behaviour depend upon the choice of point?

Possible extension

- How does this approach relate and compare to the gradients-of-chords approach?

A much more challenging problem is to try to understand the pretty curves formed by the lines on the mapping diagram. These curves become much more obvious when more lines are drawn (for example, using the GeoGebra applet). An enjoyable exploration is to look at the curves formed by different functions.