Zooming in

Teacher notes

Why use this resource?

This problem develops the idea that the gradient of a curve can be understood by zooming in to the graph. The collected results at the end can lead to a formula for the gradient function (derivative) of x^2.

Essentially, differentiation is a local phenomenon, that is, it describes an aspect of the behaviour of a function very near a given point. Zooming in allows us to focus our attention on the local region around a given point.

While it would be easy to demonstrate this using graphing software, students gain a deeper understanding of the ideas by being faced with the challenge of joining up points which lie almost on a straight line when they think they should be drawing a curve.

Calculating the coordinates of the points and drawing the curves also offers practice in working with negative numbers and rounding decimals with several decimal places; these are skills which may be somewhat insecure.

Preparation

This task involves drawing graphs of $y = x^2$ at different scales.

Students will need graph paper, rulers, pencils and calculators. There are pre-labelled graphs available to download from the "Extras" section of the resource if wanted. (Some browsers have difficulties displaying this PDF file; in such a case, right-clicking and selecting "Save Link As…" will allow you to save it and then open it with a standalone PDF reader.)

Possible approach

- Ask what students would expect to happen if they were to "zoom in" on the graph of $y = x^2$.
- Divide the class into groups of threes or fours.
- Give each group a value of b; ideally different groups should work with different values of b. Values of b which work are: 0, ± 0.5, ± 1, ± 1.5 and ± 2. Negatives are more difficult than positives to work with, and 0 is particularly hard due to the need to plot points with tricky coordinates.
- Do highlight the importance of using the specified scale on **both** axes; there is a strong temptation to scale the graph so that it uses the whole graph paper, but then it is hard to see the effect of zooming.

The “Things you might have noticed” section offers an interactive graph which shows $y = x^2$ zoomed at different scales and centred on different points, as the students will have drawn. The “Interactive graph” section offers the ability to draw and zoom in to a function of your choice, and to show tangents and secants on the graph.

Key questions

Some key questions are offered in the resource.

Once the groups have worked out the gradients of the lines for different values of b, the teacher could collate the results for the class and ask if there is any relationship between the values of b and the gradient of the graph at $x = b$, thereby leading to the concept of a gradient function.

An interesting question might also be asked: where does the change from obviously curved to an almost straight line happen most slowly? This is related to how tightly the curve bends at that point, that is, the curvature at the point.