

This shows that the original equation is equivalent to

$$\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a^2} + \frac{c}{a} = 0.$$

Since $a \neq 0$, we can divide by a to get

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0.$$

We complete the square.

We can rewrite the right-hand side by putting it over a common denominator:

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}.$$

Consider $ax^2 + bx + c = 0$, where $a \neq 0$.

Get the squared term on one side of the equation:

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{c}{a}.$$

Subtracting $\frac{b^2}{4a^2}$ from both sides and putting the right-hand side over a common denominator gives

We can take the square root of both sides.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Since x appears only once in the equation, we can rearrange this to solve for x .

Taking account of the possibility of positive and negative square roots, we see

$$x + \frac{b}{2a} = \frac{\pm\sqrt{b^2 - 4ac}}{2a}.$$