Square spirals

Teacher notes

Why use this resource?

This resource presents diagrams that offer an interesting and concrete way for students to think about infinite geometric series. Prior knowledge of geometric sequences is not required and this task could be used to introduce geometric sequences by linking the scale factor from one shape to the next to the idea of a common ratio. Working with diagrammatic representations means that no formulae are required and with some basic geometry students can find the sum of series in the form of $\frac{1}{n}+\frac{1}{n^2}+\frac{1}{n^3}+\cdots$.

Possible approaches

The warm-up could be used as a stimulus to let students explore the mathematics they see in the diagram, or it could be made more focused by using the questions in the 'Things you might think about' toggle. This could help students to appreciate how infinite sums can have a finite answer.

Once students have attempted the problem, it is likely that some talking points will have arisen. One of which is likely to be whether the patterns in the diagrams continue infinitely or not. Discussing these ideas and sharing students' own diagrams could be a way to bring out the language and ideas that surround geometric sequences and series.

Key questions

- What is the link between each term of the sequence?
- How do your answers change if you assume this sequence continues forever?
- Do we know anything about the value of this sum? What must it be bigger than? What must it be smaller than?
- Are there other ways these sums could be represented?

Possible support

There are images contained in the solution section that may help students to notice how the shaded sequences can be thought of as a simple fraction of the whole square.

Possible extension

At the end of the solution two questions are given as possible ways to extend the problem.

- Can you use these infinite sums to find the total of others? For example, can you find the total sum of $\frac{1}{2}+\frac{1}{10}+\frac{1}{50}+\frac{1}{250}+\cdots$?
- Is it possible that other sums of fractions will sum to the same totals? For example, can you find another infinite sum that equals $\frac{1}{4}$?

If exploring the second question, students may wish to look at A puzzling pentagon.