Sum estimating

Teacher notes

Why use this resource?

This problem explores the links between integration and summation. Often when introducing integration we use sums to approximate integrals. Here the idea is turned on its head and students are asked to approximate the value of a sum using an integral. This is an opportunity for a different way of thinking about sums and integrals and also touches on the common mathematical idea of finding, and improving on, upper and lower bounds for a problem that you can't find an exact answer to.

Preparation

Students need to be familiar with integrating simple polynomial functions.

Possible approach

The warm-up is ideal for some individual thinking before sharing ideas. A focus could be put on 'convincing' by encouraging students to question each others arguments.

In the main problem students should be encouraged to draw diagrams, or sketch on the printed problem to get a visual feel for the way an integral can provide an estimate for a sum. This should help them generalise the result from the specific example given, and support thinking about functions that will provide an overestimate of the sum in the second section.

In a plenary of the problem is would be worthwhile to draw out the link between summation and integration. For example, why $\sum_1^n r^2 \approx \frac{1}{3}n^3$ seems reasonable when you think about $\int_0^n x^2\,dx$, and the difference between how they first met integrals and summation, i.e. sums being used to estimate an integral, and the task they have completed, where integrals are being used to estimate sums.

Key questions

- How does using an integral to estimate a summation, compare to what you've done before using integrals and sums?
- The result shows that $\sum_1^n r^2 \approx \frac{1}{3}n^3$. Does this seem reasonable? Why might we have expected this result?

Possible support

There are images under Suggestion toggles in both the warm-up and main problem that can be used to support students' thinking.

Possible extension

There are two review questions that link to this task. Similar ideas are used in R5602 which also looks at an inequality involving sums and integrals. R6143 builds on the knowledge that students now have, i.e. that $\sum_1^n r^2$ is a cubic function, and asks them to find the exact expression of $\frac{1}{6}n(n+1)(2n+1)$.