

$$n^5 - n$$

Teacher notes

Why use this resource?

This is a lovely problem because there are several possible approaches that students might use. Students might factorise $n^5 - n$ (linking to the difference of two squares). They might use some modular arithmetic, or ideas relating to remainders. They might use proof by induction. It also consolidates and builds on the result that the product of three consecutive numbers is always a multiple of 6.

This problem can be accessed and explored by any student armed with a calculator; indeed this will give them a good feel for what's going on. However, being able to generalise this requires some more sophisticated thinking. Students will find that if they factorise the general term then they can apply the difference of two squares in order to break it down into manageable parts that can be analysed. A basic understanding of evens, odds and consecutive numbers will then help students to come to a conclusion, although some support may be required to fully justify this.