①

This is a contradiction, so our original assumption that $\sqrt{2}$ is rational must be wrong.

②

In the prime factorisations of m^2 and n^2, 2 occurs to an even power.

③

Multiply across to get $2n^2 = m^2$.

④

But prime factorisations are unique, so 2 should appear to the same power in both $2n^2$ and m^2.

⑤

Suppose, for a contradiction, that $\sqrt{2}$ is rational.

⑥

That is, we can write $\sqrt{2} = \frac{m}{n}$ where m and n are integers and where $n \neq 0$.

⑦

In the prime factorisation of $2n^2$, 2 occurs to an odd power.

⑧

So $\sqrt{2}$ is irrational.

⑨

Squaring, we have $2 = \frac{m^2}{n^2}$.

https://undergroundmathematics.org/thinking-about-numbers/square-root-of-2-is-irrational