t for tan

Teacher notes

Why use this resource?

This shows the power of working with the right diagram: students can find various important lengths and angles, and then the deduction of the double angle formulae in terms of $\tan\theta$ follows nicely.

Possible approaches

Ask each student to spend a few minutes filling in as many lengths and angles as they can on a copy of the diagram. Students could then pair up to compare their finds and try to use them to deduce the formulae.

Alternatively, students could join together in groups working on large copies of the diagram pinned to the wall after initial individual working.

Key questions

- Can you find an angle of 2θ? (How can you be sure?)
- Would it be helpful to invent a label for an unknown?
- Can you write down all the other lengths in terms of your unknown?
- What issues arise that interfere with this argument? What happens?

Possible support

- How many triangles can we identify?
- What relationships do we know in right-angled triangles? (Encourage students to recall Pythagoras' theorem and trigonometric ratios.)
  Students might need prompting to label one side, or ONLY one side.
- Can you use only one unknown?

Possible extension

Encourage students to think about the range of values of θ for which the argument works and what happens when $\theta > 45°$. Can they adapt the argument for angles above $45°$?

This resource was originally adapted from this NRICH resource. This adaptation has been featured on the NRICH website here. You might like to look at some students' solutions that have been submitted there.