Big trig

Teacher notes

Why use this resource?

This provides an opportunity for students to consider how the sine and cosine functions behave in a "which is bigger?" context. This problem is an excellent opportunity for students to think about the graphical representation of functions and can be attempted by students in a variety of ways.

Working on this problem could help students understand how to look at the behaviour of trigonometric functions by focussing on behaviour such as the limited range of the sine and cosine functions and their periodic nature. Students can also be encouraged to practise using function vocabulary and notation in the setting of trigonometry.

Teachers could encourage students to think about what they know already and then what they can deduce from that.

Possible approach

Students could start by individually trying to sketch graphs of one (or both) of the functions on whiteboards. They might also start by sketching $\sin x$ and $\cos x$ and then think about what happens when they are composed with each other. Both of these sketching tasks may raise questions in students' minds about what it would be helpful to know. Following the "think, pair, share" model, after a few minutes students could compare graphs, notes and questions and work together to try to improve their sketches.

It may be interesting to allow students access to graphing software at some point whilst working on the problem, either to confirm their findings or to aid their thinking processes. As always, students should be encouraged to explain the behaviour of the functions in addition to demonstrating their solutions graphically.

This problem can also be looked at by trying to find a value where the graphs of the functions cross. Two approaches are suggested in the solution and one of these uses the important inequality $\sin x < x$ for positive x.

Key questions

- Is the proof dependent on the functions being continuous and periodic?
- How can symmetry of graphs be used to address different sections at once or extend or adapt an idea to a different set of values of x?

Possible support

Prompts such as these could be used if students are struggling to get started.

- "What do you think the highest/lowest y-value will be?"
- "What are some specific values of these functions?"
- "How could you use your knowledge of $\sin x$ and $\cos$ here?"

Possible extension

The task could be revisited when addition formulae are introduced as addition formulae provide alternative ways to prove the result.