Going round in circles

Teacher notes

Why use this resource?

The six trigonometric functions are introduced, first via similar triangles and then via the unit circle. Key identities such as $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ and the Pythagorean identities are also deduced. From the unit circle, students are asked to visualise how the functions vary as θ changes.

Possible approach

This resource can be used in pairs as there are various ways to approach the Triangles problem. Visualising how the functions vary as θ changes could make a good group task. The interactive graphs in the Things you might have noticed section could be used to illustrate the variation and to support discussion. It might be helpful to have some graphs of the reciprocal functions available, so that students can be asked to explain how these graphs relate to the behaviour seen in the interactive graph.

Key questions

- How are the six functions related to each other? Which properties are shared? Which properties are not shared?
- What range of values can $\sec\theta$ and $\operatorname{cosec}\theta$ give us?

Possible support

Unpick some of the key questions a little for M.

- What happens to M as θ approaches $90°$? What happens when θ increases past $90°$?
- How are the coordinates of M related when $\theta = 30°$ and $150°$? Or $210°$?
- When is the y coordinate of M changing most slowly?
- When will M not move smoothly?

What could be some similar questions for N?

Can you use these ideas to investigate the behaviour of K and L?

If students are still struggling to visualize how the points are moving (how their coordinates are changing) they could look at the GeoGebra app in the Things you might have noticed section. To support visualising how the movement of the points relates to the graphs of the functions, download this extra file which shows how the graph of $y = \tan\theta$ is traced out.

Possible extension

Ask students to form simple equations such as $\sec\theta = 2$, $\tan\theta = \cos\theta$, $\operatorname{cosec}^2\theta = 1$ and try to solve them using the identities and periodicity of the functions.

- Write down an equation that has no solutions.
- If an equation has solutions, what is the most general form of solution?

Thinking about these functions could also lead into Trig tables or Can you find... trigonometry edition.