

Three lines

Teacher notes

Why use this resource?

This resource uses vector equations of lines in three dimensions. Students will practise finding intersections of lines and using Pythagoras' Theorem, and will start to appreciate how these processes are different in 3D from in 2D.

Possible approach

Encourage students to think about the equations and visualise the three lines before doing any calculations. They may conjecture or guess about the ways the lines interact, and some of these suggestions could be shared with the class. Don't allow them too long for this as not all the suggested properties can be found without calculation.

Students could then work in pairs finding points of intersection and working out what is happening geometrically. Their attention could be drawn to issues such as "three equations and two unknowns", and they could be encouraged to work out as much as possible about the resulting shape.

The [interactive GeoGebra applet](#) can be used to help visualise the individual lines, and later to illustrate the resulting geometry.

Key questions

- What do the two parts of each line equation tell us?
- A pair of lines in 2D will always intersect unless they are parallel. How does that work in 3D?
- How could we tell from the equations if these two lines pass through a common point?
- What would it mean if α and β didn't work in the third simultaneous equation?
- What shape have we got? Can you say any more about it?

Possible support

The GeoGebra applet can be used to aid visualisation of lines in 3D.

If students are struggling with the manipulation of vectors in 3D they could first use [Lots of vector lines](#) which uses similar techniques in 2D.

Possible extension

This resource could be used to introduce the scalar or dot product as a way of calculating angles between vectors.

Students could be asked to invent sets of three vector lines with given properties. For instance

- which all pass through the point $(3, 1, 2)$, or
- where two pairs intersect but the third pair do not, or
- are all perpendicular, or ...