Vector squares

Teacher notes

Why use this resource?

In this problem, students are required to interpret and work with vector equations of a line in 2D, including finding points of intersection. The problem also encourages students to make connections with geometric representations and to begin to recognise features of perpendicular lines in vector form.

This problem is very similar to Simultaneous squares which students may have encountered previously.

Preparation

This problem can be used to introduce the vector equation of a line in 2D, but is also useful for students who have already encountered it. The two approaches below might help you to think about how to use the problem with each type of student.

Possible approaches

If using this resource to introduce the vector equation of a line in 2D you might choose to present this problem alongside Simultaneous squares. The original four equations in Simultaneous squares (given in Cartesian form) are equivalent to the four vector equations presented in this resource. Students could be asked to compare the two forms and suggest alternative vector equations to represent equivalent lines before solving the problem to find the area of the enclosed square.

If using this resource with students who have already met the vector equation of a line, you might like to first ask them to convince themselves that the four equations enclose a quadrilateral, (confirming that it is a square will require deeper thinking and can be drawn out when reflecting on possible approaches to finding the area of the square – see Solution).

In both cases, allowing students a short time to look at the problem individually before encouraging them to discuss strategies in pairs or small groups can be useful. It can also be helpful to plan regular points in the lesson to stop students and ask them to think about what they are trying to find, what they have already found and whether they want to continue or change their strategies.

Key questions

- Can you represent this problem on a coordinate grid?
- Where are the vertices of the square? Do we need to know them all?
- Which lines are parallel to each other?
- Can you predict which lines will intersect each other?
- How can you find the side length of the square? Is there more than one way to do this?

Possible support

Encouraging students to actually draw the four equations on a coordinate grid can create something concrete for them to grapple with. Some students may begin by transforming the vector equations into the more familiar Cartesian form and working with them that way. You might like to allow students to do this for a short time but then spend some time reflecting with them, as an individual, group or even as a whole class, on how working in the vector form directly can be more efficient.

It is worth noting that if students construct the lines from the vector equations directly they are perhaps more likely to notice that the vertices of the square are located at fairly 'nice' positions that do not necessarily need to be calculated using simultaneous equations!

Possible extension

Another idea explored in Simultaneous squares is presented in Taking it further. This provides students with just three of the four equations that enclose a square. They must find the fourth equation and the area of the square.

Students could also be asked to create their own version of the problem for others to solve.